

INTRODUCTION

1.1 Introduction

General relativity is the theory of space, time, and gravitation formulated by Einstein in 1915. It is often regarded as a very abstruse and difficult theory, partly because the new viewpoint it introduced on the nature of space and time takes some effort to get used to since it goes against some deeply ingrained, intuitive notions, and partly because the mathematics required for a precise formulation of the ideas and equations of general relativity (namely, differential geometry) is not familiar to most physicists. Although it has been universally acknowledged as being a beautiful theory, the potential relevance of general relativity to the rest of physics has not been universally acknowledged and, indeed, probably for this reason, the subject has lain nearly dormant during much of its history.

Strong interest in general relativity began to be revived starting in the late 1950s, particularly by the Princeton group led by John Wheeler and the London group led by Herman Bondi. Although it is difficult to determine the reasons for trends in physics, two developments—relating general relativity to other areas of physics and astronomy—have contributed greatly to the sustained interest in general relativity which has continued since then. The first is the astronomical discovery of highly energetic, compact objects—in particular, quasars and compact X-ray sources. It is likely that gravitational collapse and/or strong gravitational fields play an important role here, and if so, general relativity would be needed to understand the structure of these objects. The modern theory of gravitational collapse, singularities, and black holes was developed beginning in the mid-1960s largely in response to this impetus.

A second factor promoting renewed interest in general relativity is the realization that although gravitation may be too weak to play an important role in laboratory experiments in particle physics, nevertheless it is of great importance to our further understanding of the laws of nature that a quantum theory of gravitation be developed. In order to make progress toward this goal, a deeper understanding of some aspects of the classical theory of gravitation—general relativity—may be needed. Interest in this program has been greatly strengthened by the prediction of quantum particle creation in the gravitational field of a black hole, as well as by advances in the study of gauge theories in particle physics.

But even aside from the potential impact of general relativity on astronomy and

on other branches of physics, the theory in its own right makes many remarkable statements concerning the structure of space and time and the nature of the gravitational field. After one has learned the theory, one cannot help feeling that one has gained some deep insights into how nature works.

The purpose of this book is to present the theory of general relativity. We will take a more modern, geometrical viewpoint than Einstein had, and we will, of course, discuss the recent advances and developments, but the essential content of the theory is the one Einstein gave over half a century ago. We begin in this chapter by discussing the structure of space and time and the basic ideas of relativity theory from an intuitive, physical point of view. More complete introductory discussions are given by Geroch (1978a) and Wald (1977a). The remainder of this book will be devoted to making these ideas mathematically precise and exploring their consequences.

1.2 Space and Time in Prerelativity Physics and in Special Relativity

Perhaps the greatest obstacle to understanding the theories of special and general relativity arises from the difficulty in realizing that a number of previously held basic assumptions about the nature of space and time are simply wrong. We begin, therefore, by spelling out some key assumptions about space and time. In both the past and modern viewpoints, space and time have at least the following structure in common. We can consider space and time (\equiv spacetime) to be a continuum composed of *events*, where each event can be thought of as a point of space at an instant of time. Furthermore, all events (or, at least, all events in a sufficiently small neighborhood of a given event) can be uniquely characterized by four numbers: in ordinary language, three numbers for the spatial position and one for the time.) As will be discussed in chapter 2, a mathematically precise statement of these ideas is that spacetime is a four-dimensional manifold.

However, prior to relativity theory it was believed that spacetime had the following additional structure: (Given an event p in spacetime, there is a natural, observer-independent notion of events occurring “at the same time” as p .) More precisely, given two events p and q , one of the following three mutually exclusive possibilities must hold: (1) It is possible, in principle, for an observer or material body to go from event q to event p , in which case one says q is to the past of p . (2) It is possible to go from p to q , in which case one says q is to the future of p . (3) It is impossible, in principle, for an observer or material body to be present at both events p and q . In prerelativity physics, events in the third category are assumed to form a three-dimensional set and define the notion of simultaneity with p , as is illustrated in Figure 1.1.

The belief that the causal structure of spacetime has the character shown in Figure 1.1 turns out to be wrong. In special relativity theory the above classification of the causal relationships between events still holds. (The crucial difference is that events in category (3) form more than a three-dimensional set; the causal relation between p and other events has the structure sketched in Figure 1.2. The events in category (3) can be further subdivided as follows: (i) Events that lie on the boundary of the set of points to the future of p . These events cannot be reached by a material particle

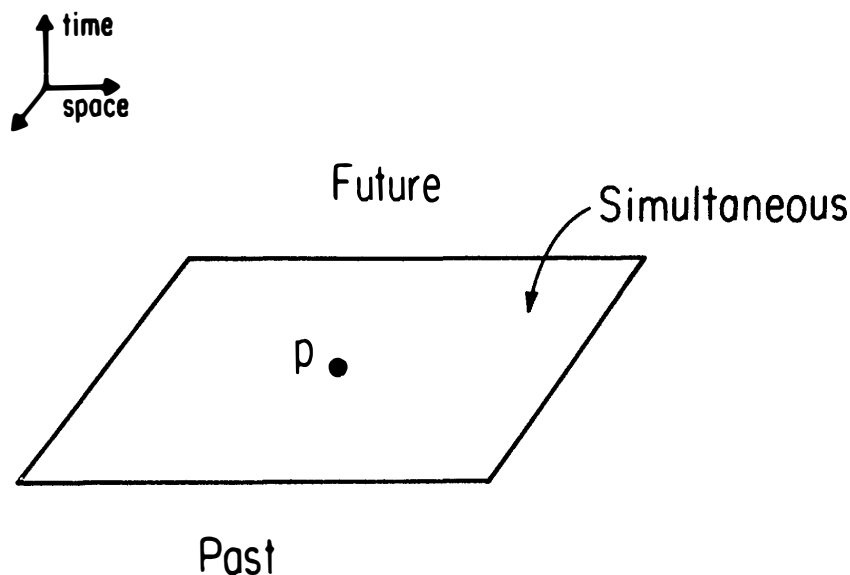
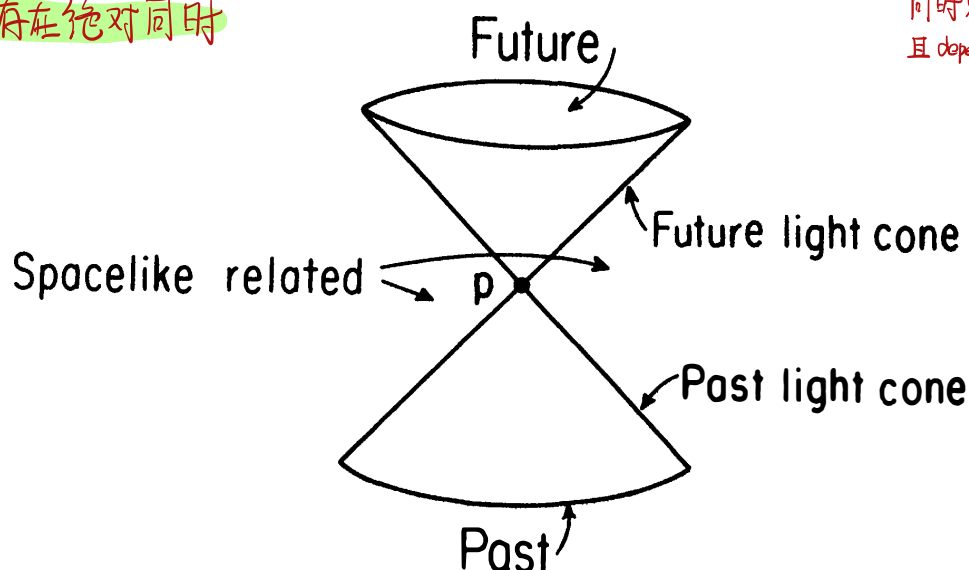


Fig. 1.1. A diagram showing the causal structure of spacetime in prerelativity physics. Given an event p , all other events in spacetime either are to the future of p , to the past of p , or simultaneous with p . The simultaneous events form a three-dimensional surface in spacetime.

starting at p but can be reached by a light signal emitted from p . They form the “future light cone” of p (a three-dimensional set). (ii) Events on the past light cone of p , defined similarly. (iii) Events in category (3) which are on neither the past nor the future light cone. These events are said to be spacelike related to p and comprise a four-dimensional set.

A key fact closely related to the above is that in special relativity there is no notion of absolute simultaneity; there are no absolute three-dimensional surfaces in spacetime as in Figure 1.1. As we shall see below, an observer still can define a notion of which events occur “at the same time” as a given event—thus defining a three-dimensional surface in spacetime—but the notion he gets depends upon his state of motion. (On the other hand, the light cones of Fig. 1.2 are absolute surfaces.) The notion that there is absolute simultaneity is a deeply ingrained one. The fact that there is no such notion is one of the most difficult ideas to adjust to in the theory of special relativity.

相对论认为不存在绝对同时



同时只能相对给定观者定义
且 depends on his state

Fig. 1.2. A diagram showing the causal structure of spacetime in special relativity. The “light cone” of p rather than a “surface of simultaneity” with p now plays a fundamental role in determining the causal relationship of p to other events.

狭义相对论承认“惯性”的概念，一个“惯性”观者可以用 rods and clocks 建立一个 global inertial coordinates

In special relativity (as in prerelativity physics) one has the notion of inertial, “nonaccelerating” motion, namely the motion a material body would undergo if subjected to no external forces. An inertial observer can label the events of spacetime in the following manner. He can build himself a rigid frame and label the grid points of the frame with the Cartesian coordinates x, y, z of the (assumed Euclidean) geometry of the frame. He can then have a clock placed at each grid point and can synchronize each clock with his by a symmetrical procedure, e.g., by making sure that a given clock and his give the same reading when they receive a signal sent out in a symmetrical manner by an observer stationed halfway between the two. (Because the causal structure of spacetime is that of Figure 1.2, not Figure 1.1, synchronization is not a trivial issue.) The observer may carry the grid, complete with synchronized clocks, in a nonrotating manner. Each event in spacetime can now be labeled with the coordinates x, y, z of the grid point at which the event occurred and the reading t of the (synchronized) clock at that event. The labels t, x, y, z assigned to events in this manner are referred to as global inertial coordinates.

If two such inertial observers go through this procedure, one may compare the coordinate labels they assign to events. In prerelativity physics (where the same labeling procedure works, the only difference being that clock synchronization is trivial), if observer O labels an event p with coordinates t, x, y, z and O' moves with velocity v in the x -direction, passing observer O at the event labeled by $t = x = y = z = 0$, the coordinate labels that O' assigns to event p are

$$t' = t, \quad (1.2.1)$$

$$x' = x - vt, \quad (1.2.2)$$

$$y' = y, \quad (1.2.3)$$

$$z' = z. \quad (1.2.4)$$

In special relativity, however, the labeling by O' will be related to that of O by a Lorentz transformation,

$$t' = (t - vx/c^2)/(1 - v^2/c^2)^{1/2}, \quad (1.2.5)$$

$$x' = (x - vt)/(1 - v^2/c^2)^{1/2}, \quad (1.2.6)$$

$$y' = y, \quad (1.2.7)$$

$$z' = z, \quad (1.2.8)$$

where c is the speed of light. Equation (1.2.5) shows that the notion of simultaneity determined by O (namely, $t = \text{constant}$) differs from that determined by O' ($t' = \text{constant}$), as illustrated in Figure 1.3.

1.3 The Spacetime Metric

In the previous section, we gave a prescription for how an inertial observer O can label the events in spacetime with global inertial coordinates t, x, y, z . However, a fundamental tenet of special relativity is that there are no preferred inertial observers. As seen above, a different inertial observer using the same procedure assigns differ-

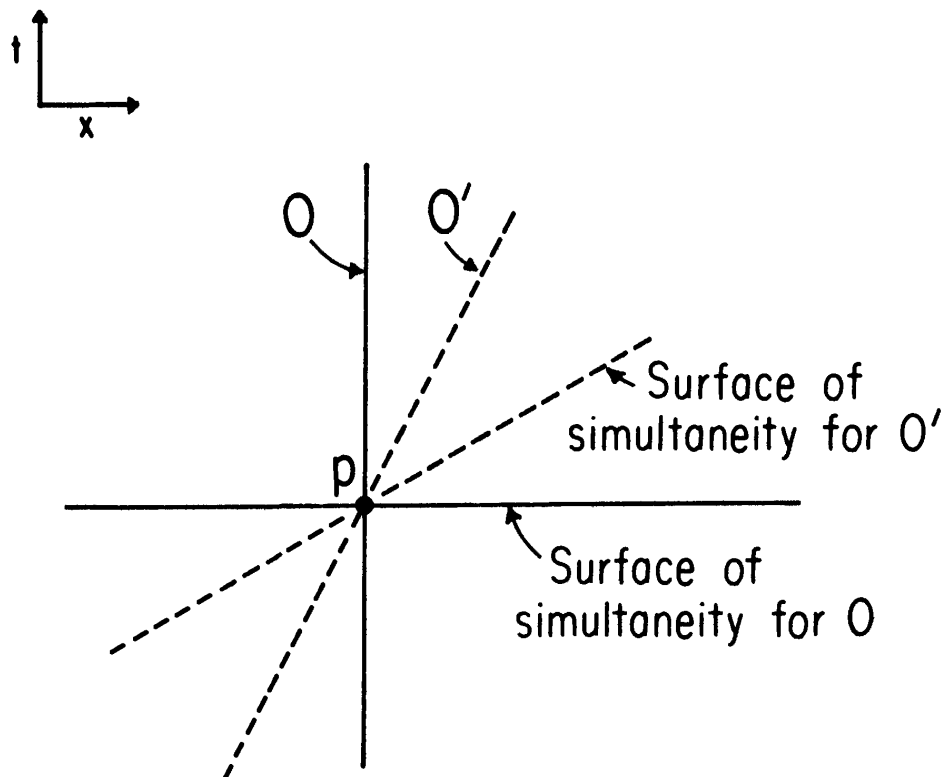


Fig. 1.3. A spacetime diagram illustrating the fact that in special relativity the inertial observers O and O' disagree over the definition of simultaneity with event p .

ent labels t', x', y', z' to the events in spacetime. Thus, the coordinate labels themselves do not have intrinsic significance since they depend as much on which observer does the labeling as they do on the properties of spacetime itself. It is of great interest to determine what quantities have absolute, observer-independent significance, i.e., truly measure intrinsic structure of spacetime. (This is equivalent to determining what functions of global inertial coordinates are independent of the choice of inertial frame.) *we are interested in* *即与选择哪套惯性标架无关的 function (though of coordinates)*

In prerelativity physics the answer is the following. The time interval Δt between two events has absolute significance; all observers will agree on the value of Δt . Furthermore, the spatial interval $|\Delta \vec{x}|$ between two simultaneous events is observer independent. However, these quantities (or functions of them) are the only ones with absolute significance. For example, observers moving with nonzero relative velocity will disagree over the spatial interval between nonsimultaneous events.

In special relativity neither the time interval nor the space interval between “relatively simultaneous” events (i.e., events determined to be simultaneous by a particular observer) has absolute significance. The quantity which is observer independent is the spacetime interval, I , defined by

$$I = -(\Delta t)^2 + \frac{1}{c^2}[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]. \quad (1.3.1)$$

定义 Poincaré 变换: global inertial coordinates 间的所有坐标变换

Indeed, the Poincaré transformations (the set of all possible transformations between global inertial coordinates) consist precisely of the linear transformations which leave I unchanged. The spacetime interval I and functions of I are the only observer-independent quantities characterizing the spacetime relationships between events.

What is truly remarkable about the expression for I is that it is quadratic in the coordinate differences, just like the distance function in Euclidean (i.e., flat, positive definite) geometry. Indeed, the only difference is the minus sign in front of $(\Delta t)^2$,

allowing I to become zero or negative. We shall refer to I as the metric of spacetime in analogy to an ordinary Euclidean metric. (More precisely, the metric of spacetime in special relativity will be defined later to be a tensor field associated with the formula for the spacetime interval between two “infinitesimally nearby” events; see eq. [4.2.2] below.) As we shall see in chapter 3, this difference in metric signature makes very little difference in the mathematical analysis of metrics. In particular definitions of geodesics (“straightest possible lines”) and curvature carry through in the same way for metrics with the signature of I as for ordinary positive definite metrics. (It is interesting to note that, as discussed more fully in chapter 4, the paths in spacetime of inertial observers in special relativity are geodesics of the spacetime metric, and the curvature associated with I is zero, i.e., the spacetime geometry in special relativity is flat.)

1.4 General Relativity

Prior to special relativity, the prerelativity notions of space and time pervaded—among many other things—the formulation of the laws of physics. When these notions were overthrown, the task remained of modifying and reformulating physical laws to be consistent with the spacetime structure given by the theory of special relativity. Maxwell’s theory of electromagnetism was already consistent with special relativity. Indeed, its incompatibility with prerelativity notions of spacetime structure unless preferred inertial frames were introduced led directly to the discovery of special relativity. Newton’s theory of gravitation is not consistent with special relativity (since it invokes notions of instantaneous influence of one body on another, but it might be thought that one could simply modify it to fit within the framework of special relativity.)

However, two key ideas motivated Einstein *not* to follow this path but rather to seek an entirely new theory of spacetime and gravitation—a theory that revolutionized our notions of space and time every bit as much as special relativity already had done.

The first idea is that all bodies are influenced by gravity and, indeed, all bodies fall precisely the same way in a gravitational field. This fact, known as the equivalence principle, is expressed in the Newtonian theory of gravitation by the statement that the gravitational force on a body is proportional to its inertial mass. Because motion is independent of the nature of the bodies, the paths of freely falling bodies define a preferred set of curves in spacetime just as in special relativity the paths in spacetime of inertial bodies define a preferred set of curves, independent of the nature of the bodies. (This suggests the possibility of ascribing properties of the gravitational field to the structure of spacetime itself.) As already mentioned in the previous section, the paths of inertial bodies in special relativity are geodesics of the spacetime metric. Perhaps, then, the paths of freely falling bodies are always geodesics, but the spacetime metric is not always that given by special relativity. What we think of as a gravitational field would then not be a new field at all, but rather would correspond to a deviation of the spacetime geometry from the flat geometry of special relativity. We shall discuss these ideas further in chapter 4.

geodesic
 in flat spacetime \Rightarrow inertial motion
 |
 in gravitational “field” \Rightarrow free fall
 ↓
 gravitation = deviation of spacetime from a flat one
 regardless of object

Mach认为 local 的 "inertial", "non-rotating" 的概念必须由 global 的 matter distribution 来定义

The second much less precise set of ideas which motivated the formulation of general relativity goes **under the name of Mach's principle**. In special relativity as in prerelativity notions of spacetime, the structure of spacetime is given once and for all and is unaffected by the material bodies that may be present. In particular, "inertial motion" and "nonrotating" are not influenced by matter in the universe. Mach as well as a number of earlier philosophers and scholars (in particular, Riemann) found this idea unsatisfactory. Rather, **Mach felt that all matter in the universe should contribute to the local definition of "nonaccelerating" and "nonrotating"; that in a universe devoid of matter there should be no meaning to these concepts**. Einstein accepted this idea and was strongly motivated to seek a theory where, unlike special relativity, the structure of spacetime is influenced by the presence of matter.

The new theory of space, time, and gravitation—**general relativity**—proposed by Einstein **states the following**: The intrinsic, observer-independent, properties of spacetime are described by a **spacetime metric**, as in special relativity. However, the spacetime metric need not have the (flat) form it has in special relativity. Indeed, **curvature**, i.e., the deviation of the spacetime metric from flatness, accounts for the physical effects usually ascribed to a gravitational field. Furthermore, the curvature of spacetime is related to the **stress-energy-momentum tensor of the matter** in spacetime via an equation postulated by Einstein. In this way, the structure of spacetime (as embodied in the spacetime metric) is related to the matter content of spacetime, in accordance with some (but not all!) of Mach's ideas. Thus far, the predictions of general relativity have been found to be in excellent agreement with experiments and observations (see section 6.3 below and Will 1981). **注意, 场方程是 postulated!**

Most of the remainder of this book is devoted to exploring the consequences of this theory. Our first task, however, is to give a precise, mathematical expression to the ideas discussed in this chapter. To begin with, we must give a precise formulation of the notion that spacetime is a four-dimensional continuum. This will be accomplished with the definition of a manifold given in section 2.1. We must then introduce the basic mathematical framework needed to discuss curved geometry: vectors and tensors (2.2), the metric (2.3), derivative operators (3.1), curvature (3.2), and geodesics (3.3). Almost all of the discussion we shall give applies equally well to the differential geometry of ordinary surfaces (positive definite metric) as to the geometry of spacetime (metric of Lorentz signature). After development of these mathematical tools and techniques, we will then be in position to begin our study of general relativity in chapter 4.

Problem

1. **Car and garage paradox**: The lack of a notion of absolute simultaneity in special relativity leads to many supposed paradoxes. One of the most famous of these involves a car and a garage of equal proper length. The driver speeds toward the garage, and a doorman at the garage is instructed to slam the door shut as soon as the back end of the car enters the garage. According to the doorman, "the car Lorentz contracted and easily fitted into the garage when I slammed the door." According to

the driver, “the garage Lorentz contracted and was too small for the car when I entered the garage.” Draw a spacetime diagram showing the above events and explain what really happens. Is the doorman’s statement correct? Is the driver’s statement correct? For definiteness, assume that the car crashes through the back wall of the garage without stopping or slowing down.